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Intro to theoretical comprehension of TNSA and modelling

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Intensive School
Laser, Plasma & Fusion
Rethymnon, Crete, Greece, 3 September 2024

Outline of the lecture

- PHYSICS OF THE LASER-PLASMA BASED ION ACCELERATION:
  ion acceleration mechanism: general features

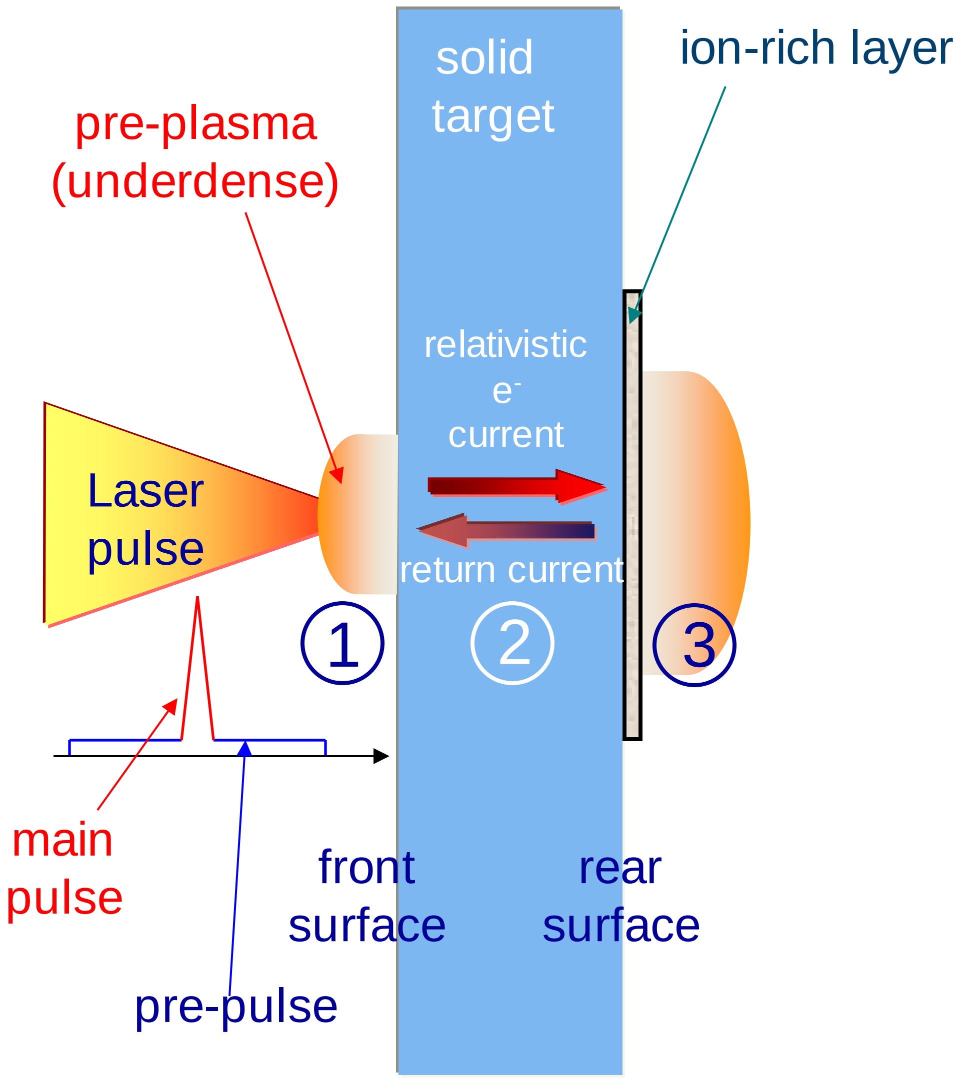
- ROLE OF THE e- POPULATION GENERATED BY THE LASER-SOLID INTERACTION:
  TNSA vs RPA vs OTHERS

- THEORETICAL MODELING OF THE TNSA ACCELERATION PROCESS:
  - quasi-neutral, self-similar plasma expansion
  - effects of non-neutrality
  - hydrodynamic models for TNSA

- "QUASI-STATIC" APPROACH TO DESCRIBE THE TNSA PROCESS:
  - introductory remarks
  - problems in using Boltzmann relation in TNSA
  - self-consistent approach: role of "bound" electrons and kinetic description of e^-
  - 2T effects: role of "cold" bulk electrons

- MODELING WITH NUMERICAL SIMULATIONS

Ion acceleration mechanism



1. **laser pulse-front surface interaction**
2. **electron propagation in the target**
3. **effective charge separation**

Protons: bulk (CH) and/or contamination layers (oils, water vapor…)
and/or coated layers

Heavy ions: bulk and/or coated layers

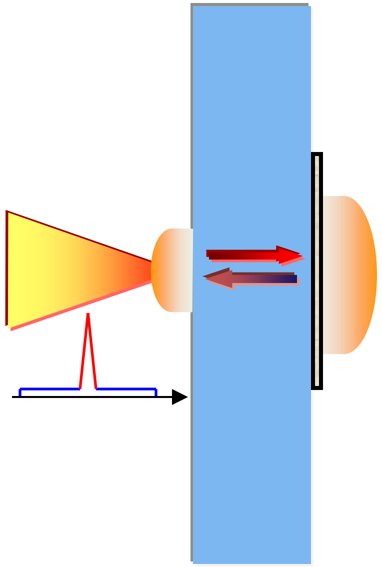
Ion acceleration mechanism

SOME CRUCIAL ISSUES

1: laser pulse-front surface interaction
- radiation pressure-driven charge separation (& resulting ion acceleration)
- possible generation of: “hot” e^-, waves, shocks…
- role of pulse properties (intensity, energy, prepulse, inc. angle, polarization)
- role of target properties (density, profile, thickness, mass)

2: electron propagation in the target
- role of electron properties (max. energy, spectrum, temperature)
- role of target properties
- return current

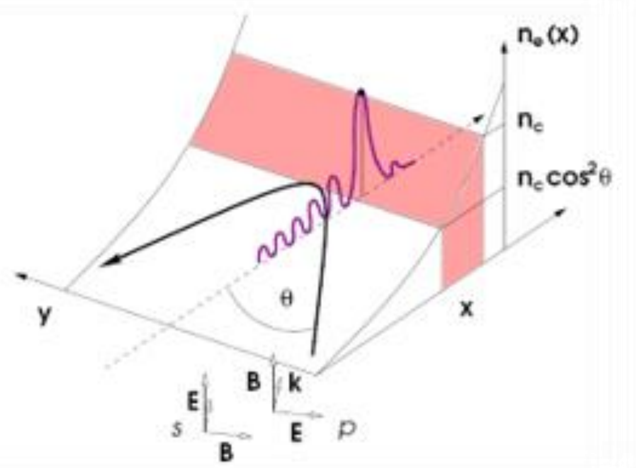
3: effective charge separation
- generation of intense electric fields
- resulting ion acceleration



SOME OPEN QUESTIONS

- which is the most effective laser absorption process at the target front?
- what is the role of each laser and target parameter?
- role of pre-pulse/pre-plasma?
- differences between front and rear acceleration?
- how to describe the acceleration process theoretically?
    - numerical simulations
    - analytical models

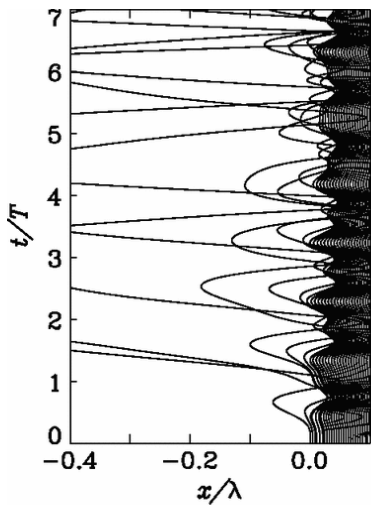
Laser-solid interactions & creation of relativistic electrons



Which are the dominant interaction mechanisms between laser and overdense plasmas at ultra-high intensities?

- collisional
- resonant absorption (N.G. Denisov, *Sov. Phys. JETP*, **4**, 544 (1957))
- Brunel effect (F. Brunel, *Phys. Rev .lett.*, **59**, 52 (1987))
- ponderomotive electron acceleration
- “J x B” heating (W. L. Kruer et al., *Phys. Fluids*, **28**, 430 (1985))

most dominant at ultrahigh intensities and ultrashort duration



For an introduction to these subjects:

- W. L. Kruer, *The physics of laser plasma interaction* (Westview Press, 1988)
- •- A. Macchi A superintense laser-plasma interaction theory primer. (Springer 2013)
- P. Gibbon, *Short pulse laser interactions with matter* (Imperial College Press, 2005)
- P. Mulser & D. Bauer, *High Power Laser-Matter Interaction* (Springer, 2010)

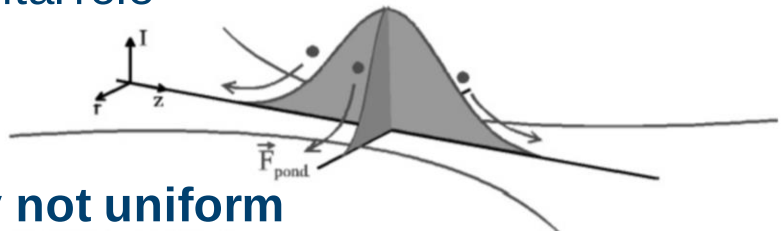
…and references therein!

! See Papadogiannis' lecture

Laser-solid interaction: ponderomotive force & JxB heating

The non-linear features of the interaction are crucial:

The so-called *ponderomotive force* plays a fundamental role



<u>What is the ponderomotive force?</u>

It is a force (acting mainly on electrons)
arising from non linear terms when the field is spatially not uniform

Difficult and subtle issue!

Within several assumptions, it can be derived computing the response at least up to E^2

- in a single particle picture (equation of motion solved iteratively)
- in a fluid description (taking into account the various non linear terms)

$$\mathbf{f}_p = -\nabla U_p$$

$$U_p \approx \begin{cases} \dfrac{e^2}{2mc^2}|\mathbf{A}|^2 & \text{non-relativistic} \\ mc^2\left(1+\dfrac{e^2}{mc^2}|\mathbf{A}|^2\right)^{1/2} - 1 & \text{relativistic} \end{cases}$$

$$(eA/m_e c^2)^2 \equiv a^2 \propto I\lambda^2$$

+ if not CP (+ normal inc.) the ponderomotive force has a 2ω oscillating term → e pulled in vacuum, accelerated within a cycle and re-injected → "**J x B heating**" occurs

B. Bezzerides et al., Phys. Rev. Lett., 44, 651 (1980)
A. Macchi et al., *Compt. Rend. Phys.*, 10, 207 (2009)

Laser-solid interaction: hot electron features

Generally speaking, because of the interaction,
laser energy is partially transferred to e$^-$ kinetic energy.

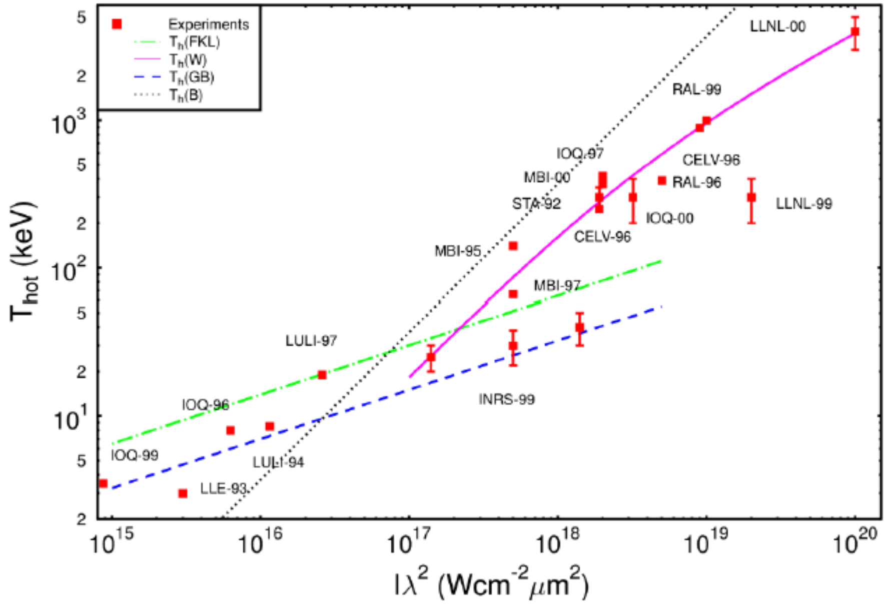
The properties of this e$^-$ population depend on:

- target properties (density, density profile)
- pulse properties (intensity, polarization)

We expect:

- charge separation (e$^-$ pushed inward)
- thermal spectrum up to high energies (Multi 10s MeV) with ultraintense LP, effective temperature $T_e \approx U_p$
  S.C. Wilks et al., *Phys. Rev. Lett*. 69, 1383 (1992)
- suppression of the thermal spectrum with CP + normal incidence
- Further phenomena in specific conditions (generation of shock waves, relativistic transparency...)

(Fig. 5.19 from Gibbon)



Collection of exp. data. Solid curve:

$$T_e \approx U_p = m_e c^2\left(\sqrt{1 + c_1 I\lambda^2} - 1\right)$$

Laser interaction with other kinds of targets

Thin solid foils are used in "conventional" (ordinary) experiments, but more advanced targets are becoming routinely employed to improve the acceleration.

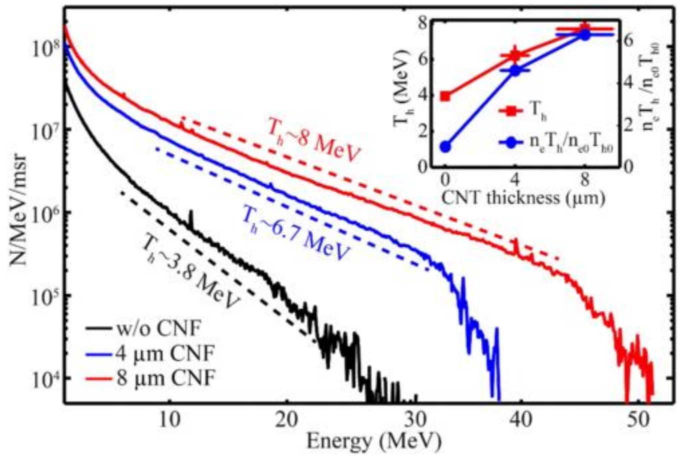
The problem of electron heating becomes even richer and more complex:

different types of targets
→ different laser-target interaction
→ different e^- heating mechanisms
→ possibly different ion acceleration physics

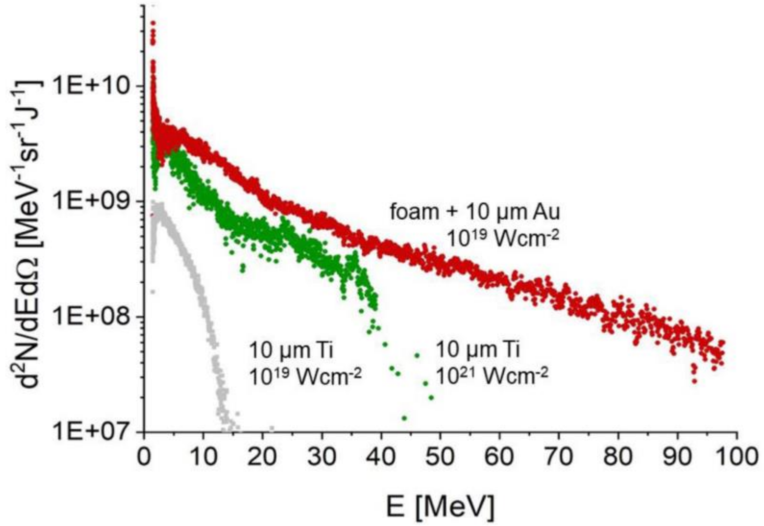
1 example:
near-critical double-layer targets:
enhanced electron heating!

But also targets that are:

- ultra-thin
- micro or nanostructured
- mass limited
- relativistically transparent

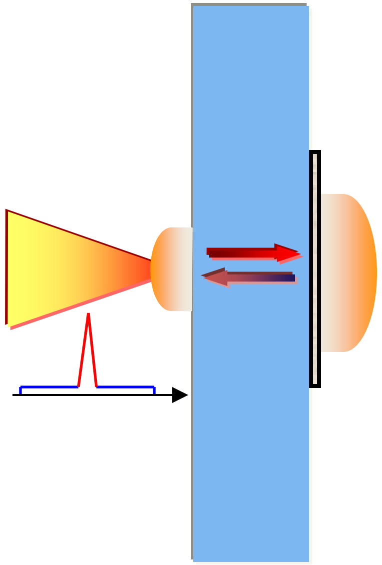


Bin, J. H., et al.
Phys. Rev. Lett. 120.7 (2018): 074801.



O. N. Rosmej et al.
Plasma Phys. Control. Fusion 62.11 (2020): 115024

Ion acceleration mechanisms: TNSA, RPA, others



IF THE e^- POPULATION IS DOMINATED BY A THERMAL SPECTRUM…
(quite “natural” physically…)

accelerating field due to charge separation between hot e^- expanding in vacuum and bulk target

Target Normal Sheath Acceleration (TNSA)

S.C. Wilks, et al., *Phys. Plasmas* 8, 542 (2001)
M. Passoni et al, *Phys.Rev. E* 69, 026411 (2004)

IF THE ROLE OF THE THERMAL e^- POPULATION IS “SUPPRESSED”
(superhigh intensity or CP + normal...is it “feasible” experimentally…?)

accelerating field induced by balance between radiation pressure and electrostatic force

Radiation Pressure Acceleration (RPA)

S.C. Esirkepov, et al., *Phys. Rev. Lett.* 92, 175003 (2004)
A. Macchi et al., *Phys. Rev. Lett.* 94, 165003 (2005)

IF LASER-TARGET PARAMETERS ARE MATCHED TO INDUCE SPECIFIC REGIMES
(laser-driven shock wave generation, relativistic induced transparency in ultrathin targets, ...)

Collisionless Shock Acceleration (CSA), Break Out Afterburner (BOA), …

Ion acceleration mechanisms: TNSA, RPA, others

IF THE e^- POPULATION IS DOMINATED BY A THERMAL SPECTRUM…
(quite "natural" experimentally…)

accelerating field due to charge separation between hot e^- expanding in vacuum and bulk target

Target Normal Sheath Acceleration (TNSA)

S.C. Wilks, et al., *Phys. Plasmas* 8, 542 (2001)
M. Passoni et al, *Phys.Rev. E* 69, 026411 (2004)

We will **focus on the TNSA regime because**:

- dominant in the most common and less stringent experimental conditions + difficult to avoid anyway
- relevant for near-future applications
- didactically interesting
- analytical, semi-analytical and numerical approaches

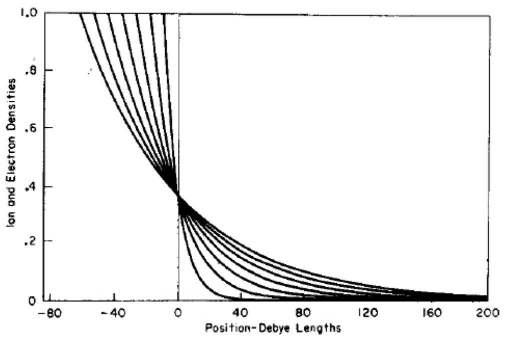
Note: most of the focus will be on the ion maximum energy and spectrum (in line with experiments)

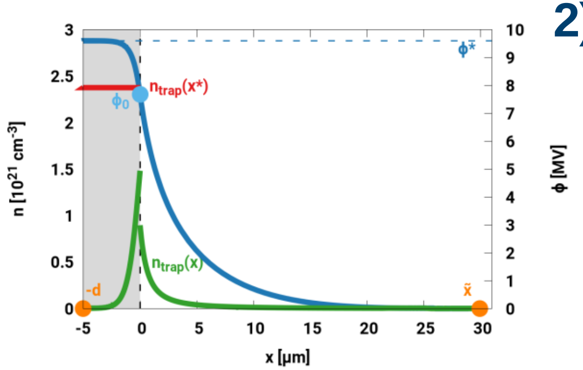
Theoretical description of laser-based ion acceleration: TNSA

Our main focus will be on the TNSA regime

(by far the dominant one in most common present exp. conditions)

How to develop analytical models
of the acceleration process in TNSA?



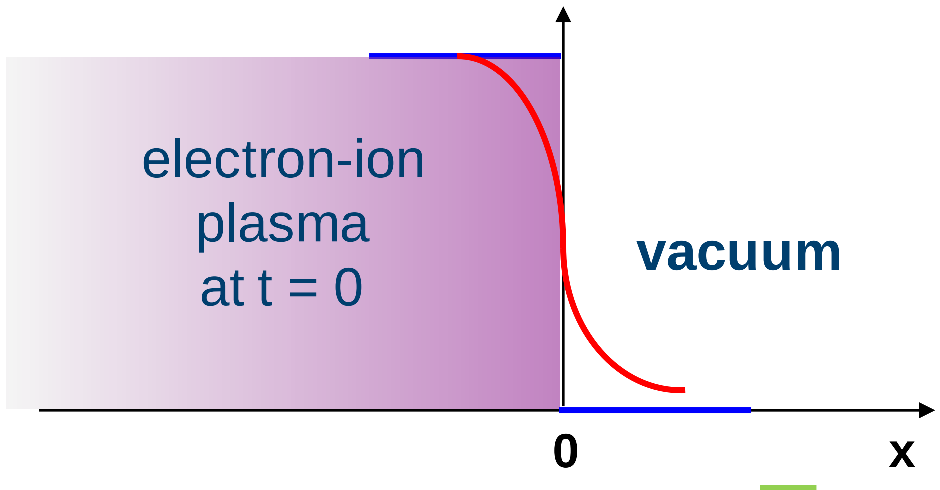


2 main “complementary” approaches have been developed

1) **PLASMA EXPANSION IN VACUUM: accelerated ions and hot electrons constitute an expanding plasma which is described with fluid or kinetic models**
   - accelerated ions are the positive component of a globally neutral plasma
   - focus on the collective time evolution of ion dynamics

2) **QUASI-STATIC MODELS: describe in detail the accelerating field as a quasi-static sheath electric field set up by the hot electrons**
   - light ions treated as test particles forming a thin low-density layer
   - heavier ions considered almost immobile on the time scales of light ion acceleration
   - focus on the early stages of ion acceleration (energetic ions)

TNSA as plasma expansion in vacuum: 1D fluid model



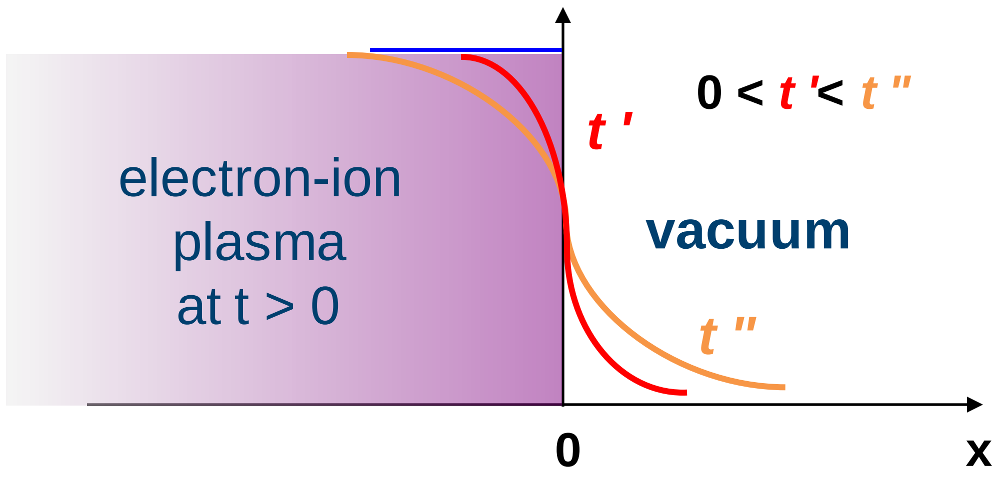
$t = 0$

- cold ions occupy a semi-infinite region (x<0)
- hot electrons, due to their thermal energy, are separated by the ions

→ an electric field is produced

$t > 0$

- hot electrons start dragging the ions due to charge separation
- the whole plasma expands in a semi-infinite vacuum



TNSA as plasma expansion in vacuum: 1D fluid model

- 1D fluid equations with $T_i = 0$ and T_e = const **isothermal model**

continuity

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0$$

momentum **no ion collisions**

$$m_i \left(\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} \right) = -Ze\frac{\partial \phi}{\partial x}$$

self-consistent electrostatic field

$$E = -\frac{\partial \phi}{\partial x}$$

- Poisson equation for the electrostatic field

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e(Zn_i - n_e)$$ **charge separation**

- Equations for the electron dynamics are required

$$n_e = n_0 \exp\left(\frac{e\phi}{T_e}\right)$$

If electrons can be assumed in equilibrium with the electrostatic potential: Boltzmann relation (very common…but see what happens!)

TNSA as plasma expansion in vacuum: 1D fluid model

introduce normalization constants

$$\bar{x} = \frac{x}{x_0}, \bar{n} = \frac{n}{n_0}, \bar{\phi} = \frac{e\phi}{T_e}$$

Poisson equation then becomes

$$\frac{\partial^2 \phi}{\partial x^2} = \boxed{\frac{x_0^2}{\lambda_D^2}}(Zn_i - n_e)$$

(overbars are removed)

there is an implicit spatial scale in the problem: the Debye length

$$\lambda_D = \sqrt{\frac{T_e}{4\pi e^2 n_0}}$$

we then set $x_0 = \lambda_D$

in general, the evolution is not "self-similar", however…

TNSA as plasma expansion in vacuum: 1D fluid model

$$t_1 = 2\pi \frac{\lambda_{De}}{v_{te}} = \frac{2\pi}{\omega_{pe}}$$

electrons and ions are separated on average by a distance ~ λ_{De}

an ambipolar electrostatic field arises, which opposes to further charge separation

$$t_2 = 2\pi \frac{\lambda_{De}}{c_s} = \frac{2\pi}{\omega_{pi}}$$

t

for $t \gg t_2$ slow expansion of the plasma as a whole at $c_s = (ZT_e/m_i)^{1/2}$

$N_e = 10^{21}$ cm^{-3}
$T_e = 1$ keV
$\longrightarrow$
$\lambda_{De} \approx 7$ nm
$v_{te} \approx 10^7$ m/s
$t_1 \approx 3 \times 10^{-15}$ s

$N_e = 10^{19}$ cm^{-3}
$T_e = 1$ keV
$\longrightarrow$
$\lambda_{De} \approx 70$ nm
$c_s \approx 3 \times 10^5$ m/s
$t_2 \approx 1.5 \times 10^{-12}$ s

In the ps-ns regime:

- the typical scale of the plasma inhomogeneity becomes $c_s t \gg \lambda_{De}$
- the plasma maintains quasi-neutrality $|Zn_i - n_e|/n_0 \ll 1$
- plasma moves "slowly", at $c_s = (ZT_e/m_i)^{1/2}$
- electrons have time to reach an equilibrium

therefore, we can close the system using $\longrightarrow$

$$\begin{cases} n_e = n_0 \exp\left(\dfrac{e\phi}{T_e}\right) \\ Zn_i \approx n_e \end{cases} \longrightarrow \frac{e\phi}{T_e} = \ln\frac{n_e}{n_0} \approx \ln\frac{Zn_i}{n_0}$$

TNSA as plasma expansion in vacuum: 1D fluid model

Assuming

- quasi-neutrality: $n_e = Zn_i$ (strictly valid only for times $t \gg t_2$)
- cold ions: $T_i = 0$
- hot electrons in thermal equilibrium $n_e = n_0 \exp(e\phi/T_e)$

then the expansion is governed by the system

$$\begin{cases} \dfrac{\partial n_i}{\partial t} + \dfrac{\partial}{\partial x}(n_i v_i) = 0 \\ m_i \left(\dfrac{\partial v_i}{\partial t} + v_i \dfrac{\partial v_i}{\partial x} \right) = -ZT_e \dfrac{\partial}{\partial x} \dfrac{\ln n_i}{n_0} \end{cases}$$

"*Self-similar motion of rarefied plasma*"
A.V. Gurevich, *et al.*, *Sov. Phys. JETP* 22, 449 (1966) → (also kinetic!)

Formal and physical analogy with the problem of the expansion of a 1D ideal fluid in a semi-infinite space with

$$c_s = \sqrt{\frac{ZT_e}{m_i}}$$ ion sound speed

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = 0 \\ \dfrac{\partial v}{\partial t} + v \dfrac{\partial v}{\partial x} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = -c_s^2 \dfrac{1}{\rho}\dfrac{\partial \rho}{\partial x} \end{cases}$$

$$c_s = \sqrt{\frac{\partial p}{\partial \rho}}$$ sound velocity of the fluid

TNSA as plasma expansion in vacuum: 1D fluid model

Such a system has fundamental properties:
- no characteristic length or time
- characteristic velocity (sound velocity of the system, c_s)

Therefore, an isothermal quasi-neutral plasma is characterized by a self-similar expansion

It admits a "self-similar" solution
- all quantities [$\rho(x,t)$, $v(x,t)$] must depend on x/t
- the spatial distribution of all quantities at various instants is similar, differing only in the scale which increases with time

General technique of solution for this system:

introducing $\xi = x / t$ and observing that

$$\frac{\partial}{\partial x} = \frac{1}{t}\frac{d}{d\xi}, \qquad \frac{\partial}{\partial t} = -\frac{\xi}{t}\frac{d}{d\xi}$$

the equations become

$$(v-\xi)\rho' + \rho v' = 0$$

$$(v-\xi)v' = -c_s^2 \rho'/\rho$$

and eliminating ρ' and v'

$$(v-\xi)^2 = c_s^2$$

$$\rho v' = -c_s \rho'$$

$$\xi = \frac{x}{t} = v - c_s$$

$$v = -\int c_s d\rho/\rho$$

choice of the sign is conventional

Landau & Lifshitz, *Fluid Mechanics*, chapter X

TNSA as plasma expansion in vacuum: 1D fluid model

In our case $\boxed{\xi = \frac{x}{t} = v - c_s}$ $\boxed{v = -\int c_s d\rho/\rho}$ become $\boxed{\frac{x}{t} = v_i - c_s}$ $\boxed{v_i = -c_s \int \frac{dn_i}{n_i}}$

(c_s is constant)

For $x > -c_s t$, then **rarefaction wave**:

$$n_i = n_0 \exp(-v_i/c_s) = n_0 \exp[-1 - x/(c_s t)]$$

$$v_i = c_s + \frac{x}{t}$$

$$\frac{e\phi}{T_e} = \frac{\ln n_i}{n_0} = -\frac{x}{c_s t}$$

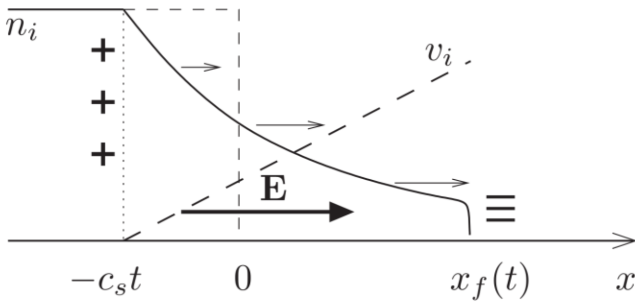
$$E = \frac{T_e}{e c_s t} = \frac{T_e}{e L_n}$$

accelerating field

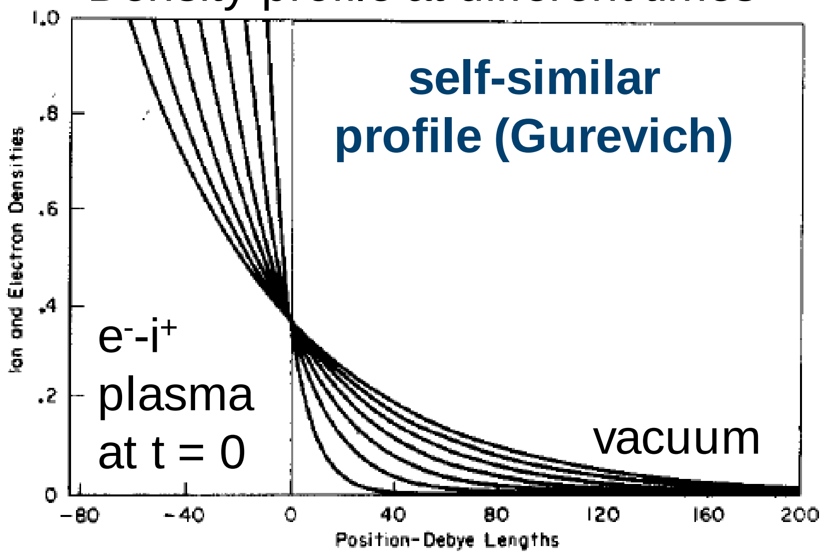
$$L_n = \frac{n_i}{|\partial n_i/\partial x|} = c_s t$$

local plasma scale length

Sketch of the solution



Density profile at different times



Note: this solution has several divergences, which require more physical considerations to be properly interpreted…

S.C. Wilks, *et al.*, *Phys. Plasmas* 8, 542 (2001)

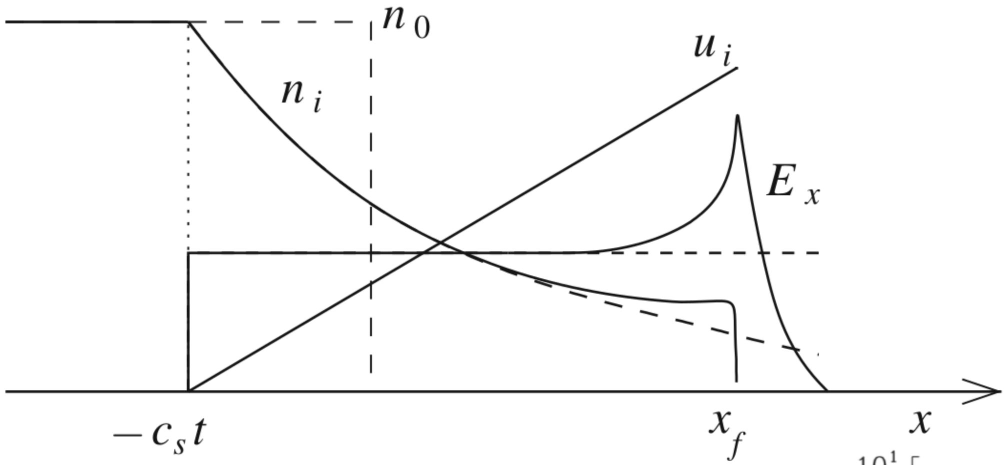
Landau & Lifshitz, *Fluid Mechanics*, chapter X

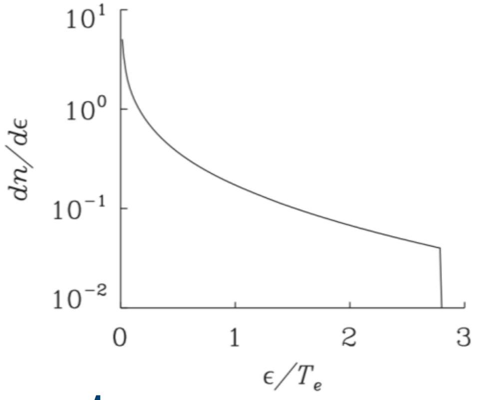
TNSA as plasma expansion in vacuum: 1D fluid model

For x > - c_st, then **rarefaction wave**:

$$\left\{\begin{array}{l} n_i = n_0 \exp[-1 - x/(c_s t)] \\ v_i = c_s + \dfrac{x}{t} \longrightarrow \mathcal{E}_i = \dfrac{1}{2} m_i v_i^2 \\ E = \dfrac{T_e}{e c_s t} = \dfrac{T_e}{e L_n} \end{array}\right.$$

Sketch of the solution





v_i increases indefinitely with x
→ there is a maximum x for which the Q. N. solution makes sense and it is given by the following condition

$$L_n = \lambda_D(n_e(x/t))$$

breaking of neutrality

ion front

$$x_f(t) = c_s t[2\ln(\omega_{pi} t) - 1]$$

$$v_f(t) = v_i(x_f(t)/t)$$

$$\mathcal{E}_{i,max} = \frac{1}{2} m_i v_f^2$$

ion spectrum

$$\frac{dn_i}{d\mathcal{E}_i} = \frac{n_0}{\sqrt{2\mathcal{E}_i T_e}} \exp\left(-\sqrt{\frac{2\mathcal{E}_i}{T_e}}\right)$$

A. Macchi *A superintense laser-plasma interaction theory primer*. Springer Science & Business Media, 2013.

Comments on quasi-neutral dynamics

- self-similar plasma expansion takes place for $t \gg t_2$

(ok for pre-CPA experiments → "long" laser pulses ≈ ns scale)

- the motion consists in an unsteady rarefaction wave

(plasma expansion into vacuum over times >> $2\pi/\omega_{pi}$)

- the residual electric field E accelerates part of the ions flowing into vacuum $E = \frac{T_e}{eL_n}$
- the ion motion is characterized by

$$n_i = n_0 \exp\left[-\left(1 + \frac{x}{c_s t}\right)\right] \qquad v_i = c_s + \frac{x}{t}$$

Issues of the self-similar quasi-neutral model, if applied to describe the ion acceleration process induced by ultrashort lasers

it is not valid on the time scale typical of ultrashort fs lasers:

- on the relevant time scales, $L_n = c_s t \ll \lambda_D$ quasi-neutrality brakes down!
- the typical scale length of the accelerating field becomes λ_D: a so-called **Debye sheath** is formed

we must go beyond our starting assumptions……

TNSA ion acceleration mechanism

Estimate of the field at the rear surface from Q.N. analysis → $E_{QN} = \frac{T_e}{eL_n}$ $L_n = c_s t$

(t ~ ps – ns)

If charge separation is relevant:

- Order of magnitude of electron energy → T_e
- Order of magnitude of electron cloud extension → λ_{De}

→ $E = \frac{T_e}{e\lambda_{De}}$

Estimate T_e assuming that the ponderomotive potential is the main mechanism for electron heating →

$$T_e \approx m_e c^2 \left[\sqrt{1 + \frac{I\lambda^2(\mu\mathrm{m})}{1.4 \times 10^{18} \mathrm{W\ cm^{-2}}}} - 1 \right]$$

For ultraintense ultrashort laser pulses
$I > 10^{18}$ W/cm^2, $\lambda \sim 1$ μm, $n_e \sim n_c \sim 10^{21}$ cm^{-3} →

T_e ~ MeV, t ~ 100 ps
→ λ_{De} ~ μm << L_n ~ 1 mm
→ **E ~ MV / μm >> E_{QN} ~ MV / mm**

Such E fields can accelerate ions up to several MeV on a micrometer scale length: this is the TNSA field!

S.C. Wilks, *et al*., *Phys. Plasmas* 8, 542 (2001)

Plasma expansion in vacuum with charge separation

Effects of non-neutrality in the plasma expansion

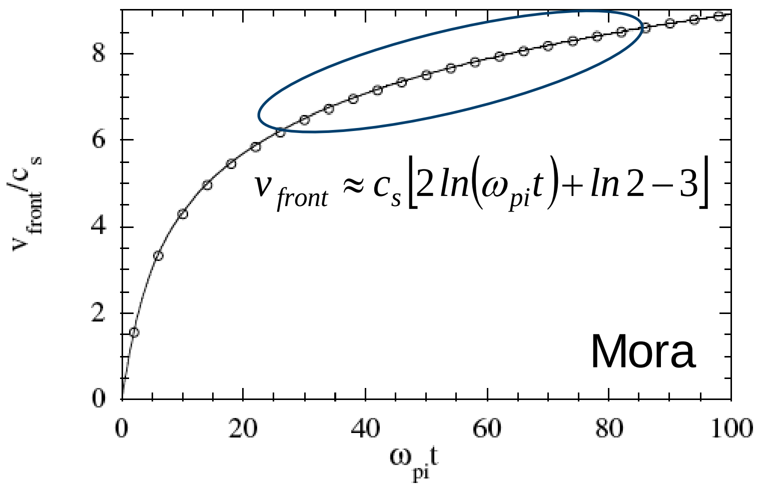
$$
\begin{cases}
\dfrac{\partial n_i}{\partial t} + \dfrac{\partial}{\partial x}(n_i v_i) = 0 \\
m_i \left(\dfrac{\partial v_i}{\partial t} + v_i \dfrac{\partial v_i}{\partial x} \right) = -Ze \dfrac{\partial \phi}{\partial x} \\
\dfrac{\partial^2 \phi}{\partial x^2} = -4\pi e (Z n_i - n_e) \\
n_e = n_0 \exp\left(\dfrac{e\phi}{T_e} \right) \\
n_i(t=0) = \dfrac{n_0}{Z} H(-x)
\end{cases}
$$

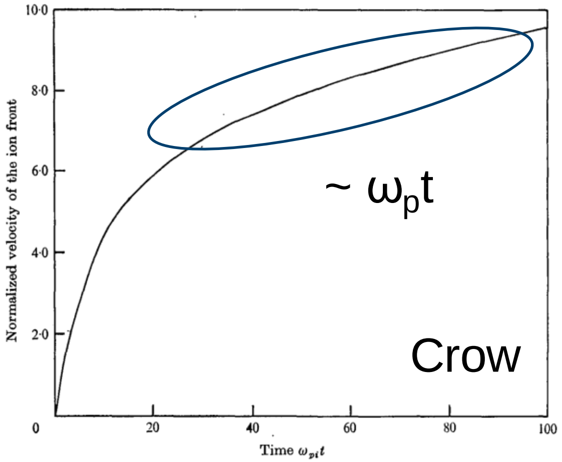
charge separation

Heaviside function

Solution via numerical integration

Velocity of the ion front in time





The front velocity still increases indefinitely in time, hence the maximum ion energy – it's a consequence of Boltzmann
→ A <u>maximum ion acceleration time</u> t_{acc} must be introduced to obtain an actual energy cut-off

"*Plasma expansion into a vacuum*" M. Widner, *et al.*, *Phys. Fluids* 14, 795 (1971)
"*The expansion of a plasma into a vacuum*" J.E. Crow, *et al.*, *J. Plasma Phys.* 14, 65 (1975)
"*Plasma expansion into a vacuum*" P. Mora, *Phys. Rev. Lett.* 90, 185002 (2003)

Mora's hydrodynamic code

- ion fluid equations ($T_i = 0$)
- Boltzmann electron distribution
- Poisson equation

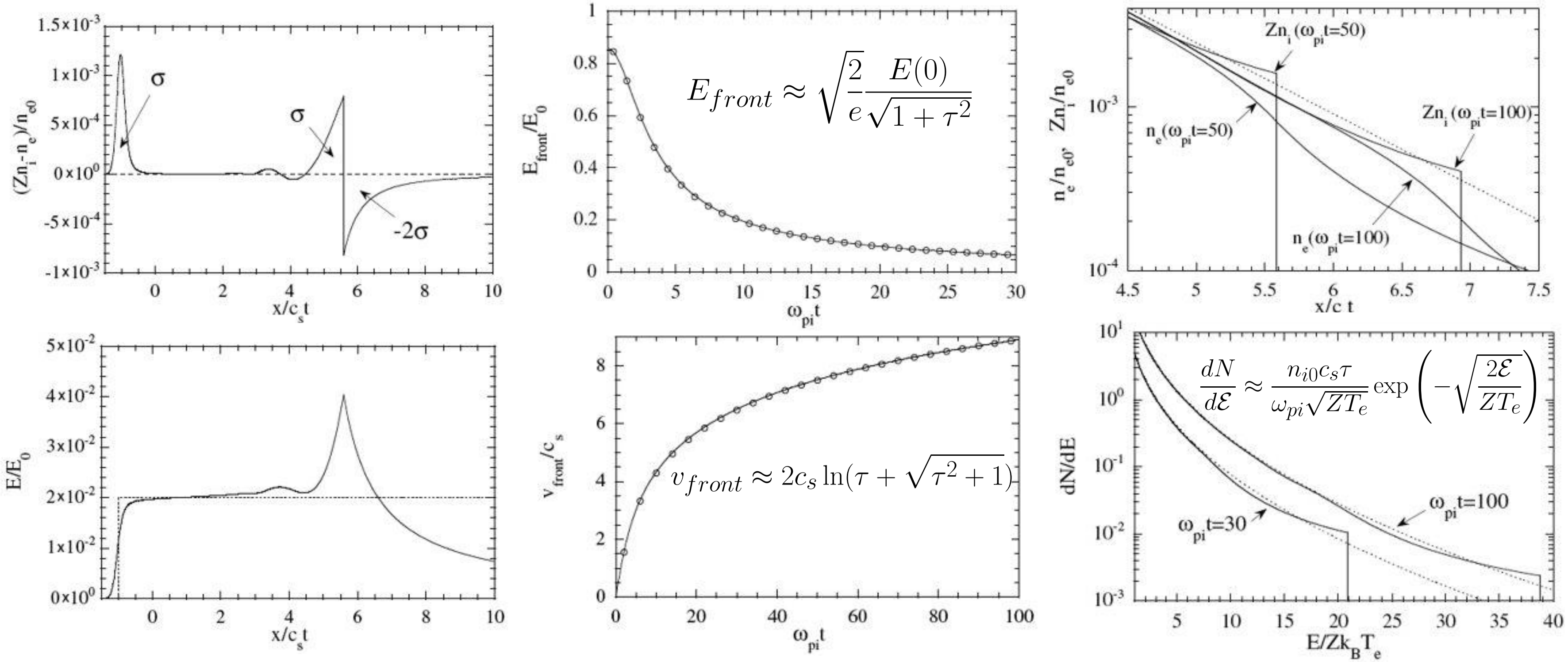
like before!

- Lagrangian code: Poisson is integrated between $x = x_{front}$ and $x = \infty$

"*Plasma expansion into a vacuum*"
P. Mora, *Phys. Rev. Lett.* **90**, 185002 (2003)

Interpolating the numerical result, the following approximate expressions for the time dependence of the physical quantities at the ion front can be obtained:

$$\tau = \frac{\omega_{pi} t}{\sqrt{2e}}$$



maximum kinetic energy at the ion front

$$\mathcal{E}_{front} \approx 2ZT_e \left[\ln\left(\tau + \sqrt{\tau^2+1}\right)\right]^2$$

$\mathcal{E}_{front} \to \infty$ as $\tau \to \infty$

(consequence of Boltzmann, see below!!)

a maximum ion acceleration times must be introduced to describe the energy cutoff

Plasma expansion in vacuum: comparison with experiments

This model has been found very useful and it has been widely adopted to interpret many TNSA experimental data. How?

1. Define model parameters:
   - $T_e \rightarrow$ ponderomotive scaling
   - $n_e \rightarrow$ physical / geometrical arguments on laser absorption
   - $t_{acc} \sim 1.3 \times$ pulse duration (fitted)
2. Compute the maximum ion energy using the model formula, e.g.

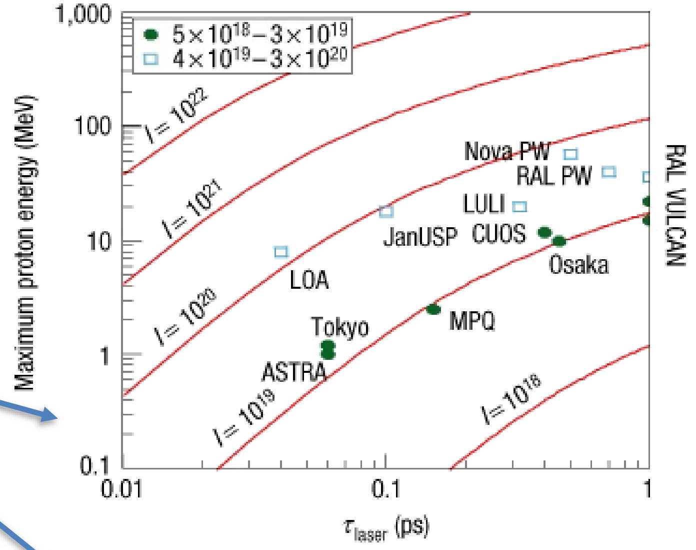
$$\mathcal{E}_{i,max} = 2T_e \left[\ln(t_p + \sqrt{t_p^2 + 1}) \right]^2$$

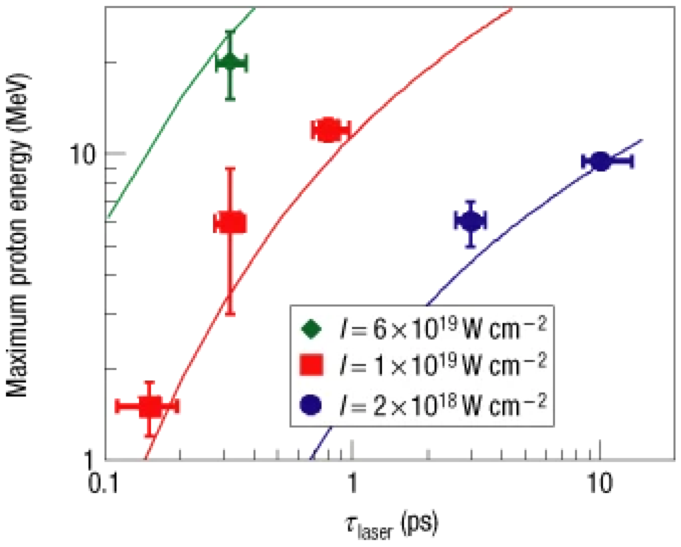
$$t_p = \frac{\omega_{pi} t_{acc}}{\sqrt{2e}}$$

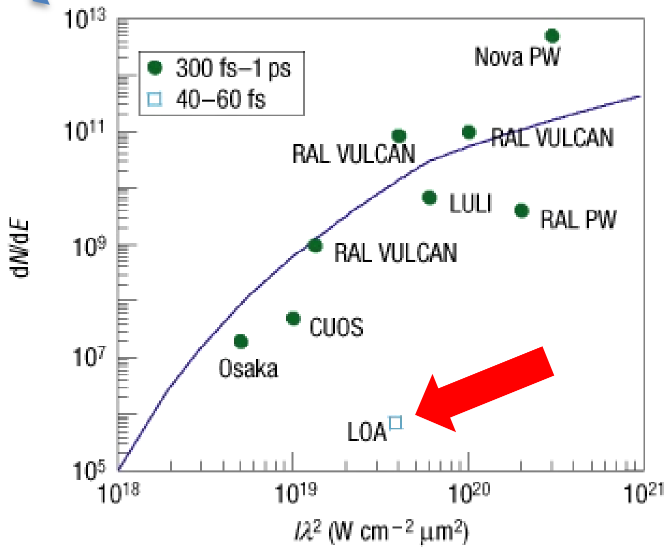
Does not work too well with ultrashort pulses (< 100 fs)

J. Fuchs, et al., Nature Phys. 2, 48 (2006)

- lines from fitted Mora model
- points from experiments







Limits of the plasma expansion in vacuum

This model has been found very useful and it is widely used to interpret many TNSA experimental data, however:

- the accelerated protons/ions are a thin layer of positive charge rather than a semi-infinite expanding plasma
- the empirical acceleration time can be unphysical in several situations, i.e. not corresponding to the actual ion acceleration time
  - too short for short pulses (tens fs)
  - too long for the most energetic part of the spectrum and “long” pulses (few ps)
- divergent maximum ion energy

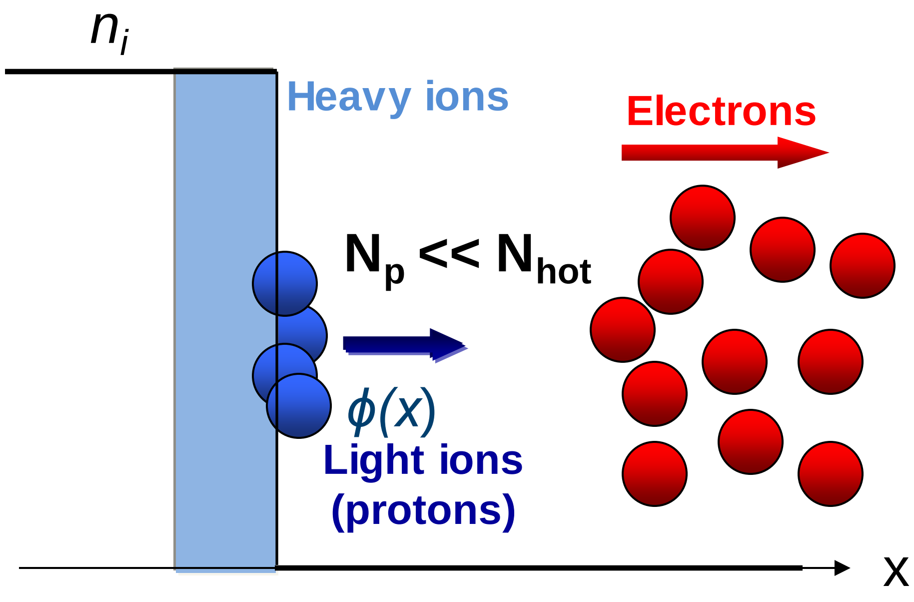
Let's consider an alternative modeling approach

Quasi-static TNSA models

Assume that

- hot electrons create a non-neutral region, source of an electric field
- light ions form a thin layer, the main target is made of heavier ions
- during the characteristic acceleration time of the light ions the **hot electrons are almost isothermal** (cooling becomes important at longer times) and the **heavier ions almost immobile**
- until the number of accelerated light ions is much lower than the number of hot electrons, the field is not heavily affected

the accelerating field can be assumed quasi-static, light ions treated as test particles



This description can be the most suitable to describe <u>the most energetic part of the ion spectrum</u>

we do not speak any more of "*plasma expansion* " but of *"particle acceleration"*

Description of the hot electrons

If we assume:

- Boltzmann density distribution of isothermal electrons

$$n_e = n_0 e^{e\phi/T_e}$$

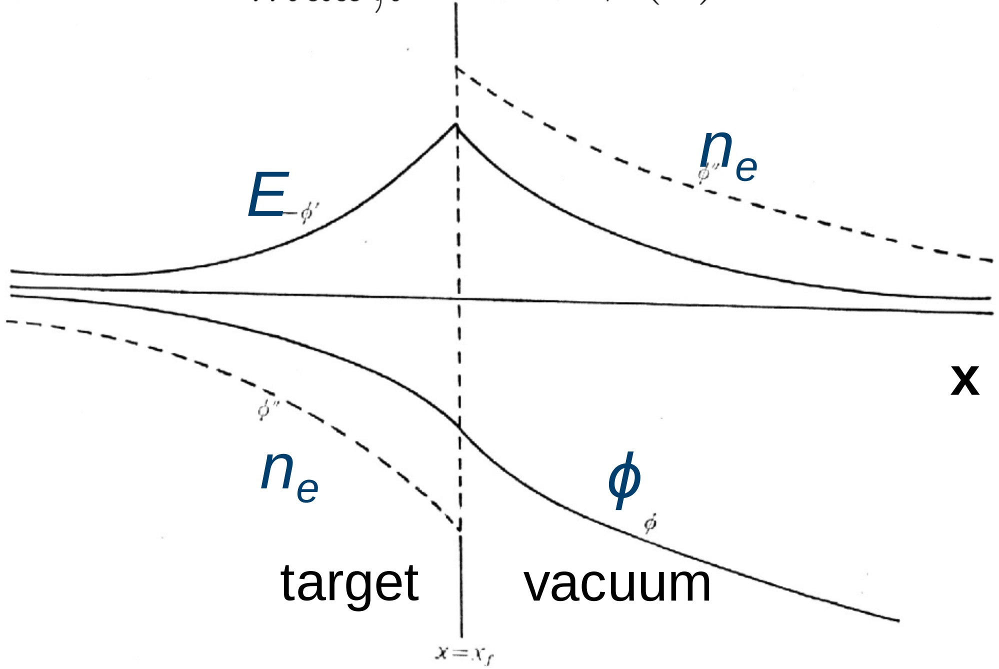
- Poisson equation extending over a semi-infinite domain

→ Exact solution for $x > 0$

$$\frac{e\phi(x)}{T_e} = -2\ln\left[1 + \frac{1}{\sqrt{2e}}\frac{x}{\lambda_D}\right] - 1$$

$$\begin{cases} \dfrac{\partial^2\phi}{\partial x^2} = 4\pi\left(n_0 e^{e\phi/T_e} - Zn_i\right) & x > 0 \\ \dfrac{\partial^2\phi}{\partial x^2} = 4\pi n_0 e^{e\phi/T_e} & x < 0 \end{cases}$$

$$\mathcal{E}_{max,i} = eZ\phi(0)$$



"Boltzmann" e⁻ in infinite space inevitably gives: $\boldsymbol{\phi}$**(x→+∞)** → **-∞** !
(in order to have $n_e(x\to+\infty)$ & $E_x(x\to+\infty)\to 0$)

A test ion initially at rest in $x = 0$, will get an infinite energy in an infinite time!

regardless the dimensionality, final ion energy diverges !!!

J.E. Crow, *et al*., *J. Plasma Phys.* 14, 65 (1975)

Description of the hot electrons

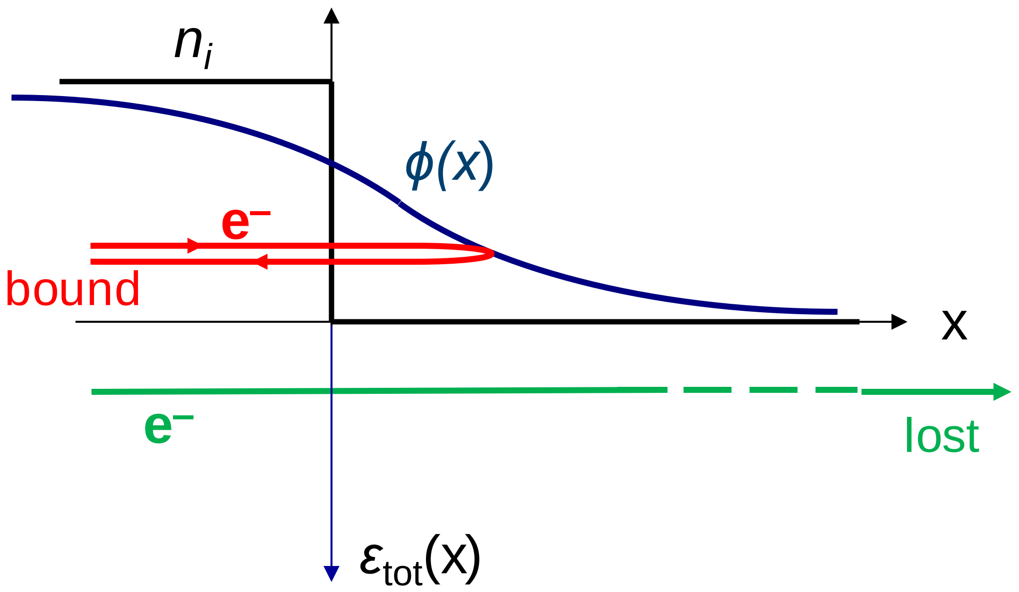
What to do then?

- remove the isothermal assumption

V.F. Kovalev, et al., *JETP,* 95, 226 (2002)
P. Mora, *Phys. Rev. E* 72, 056401 (2005); *Phys. Pl.* 12, 112102 (2005)
S. Betti, et al., *Pl. Phys. Contr. Fus*. 47, 521 (2005)

- isothermal models: introduce physical "truncation mechanisms"

Y. Kishimoto, et al., *Phys. Fluids* 26, 2308 (1983)
M.Passoni, M.Lontano, *Laser Part. Beams* 22, 171 (2004)
M. Lontano, M. Passoni, *Phys.Plasmas* 13,042102 (2006)
J. Schreiber, et al., *Phys. Rev. Lett.* 97, 045005 (2006)
M. Passoni, M. Lontano, *Phys. Rev. Lett.* 101, 115001 (2008)



- build a self-consistent approach considering the role of bound electrons

consider the electron distribution function and kinetic energy

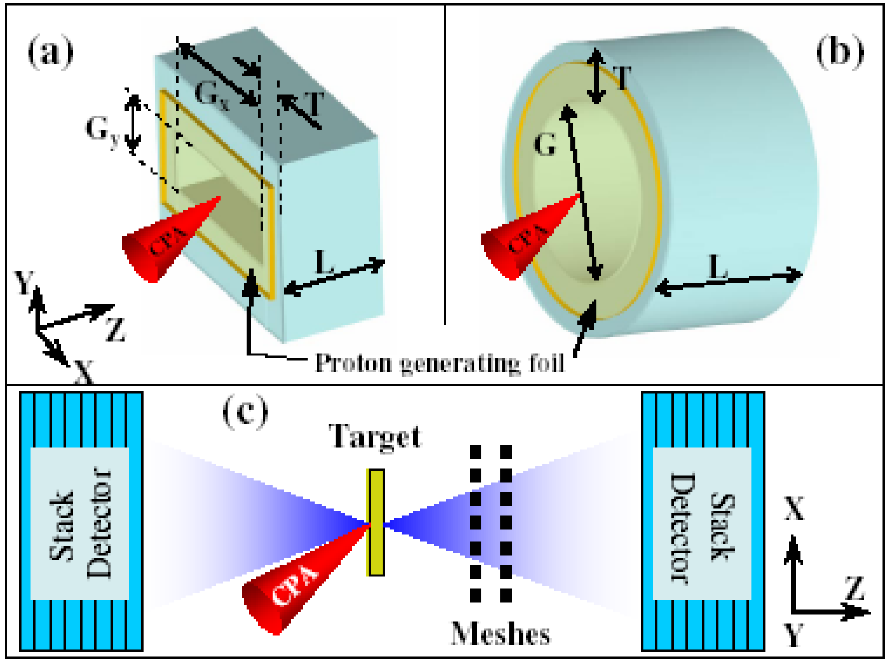
$$f_e = f_e(\mathbf{r},\mathbf{p},T_e,\phi) \qquad \varepsilon(\mathbf{r},\mathbf{p}) = mc^2\gamma - e\phi(\mathbf{r})$$

"E.S. field distribution at the sharp interface between high density matter and vacuum"
M. Lontano, M. Passoni, *Phys.Plasmas*, **13**, 042102 (2006)

- only "trapped" ($\varepsilon(\mathbf{r},\mathbf{p}) < 0$) e⁻ are bound from the potential to the target
- "passing" e⁻ ($\varepsilon(\mathbf{r},\mathbf{p}) > 0$) leave the system

Role of bound electrons

Any experimental evidence of "passing" vs "bound" electrons?



"*Dynamic Control of Laser-Produced Proton Beams*"
S. Kar et al., *Phys. Rev, Lett.*, **100**, 105004 (2008)

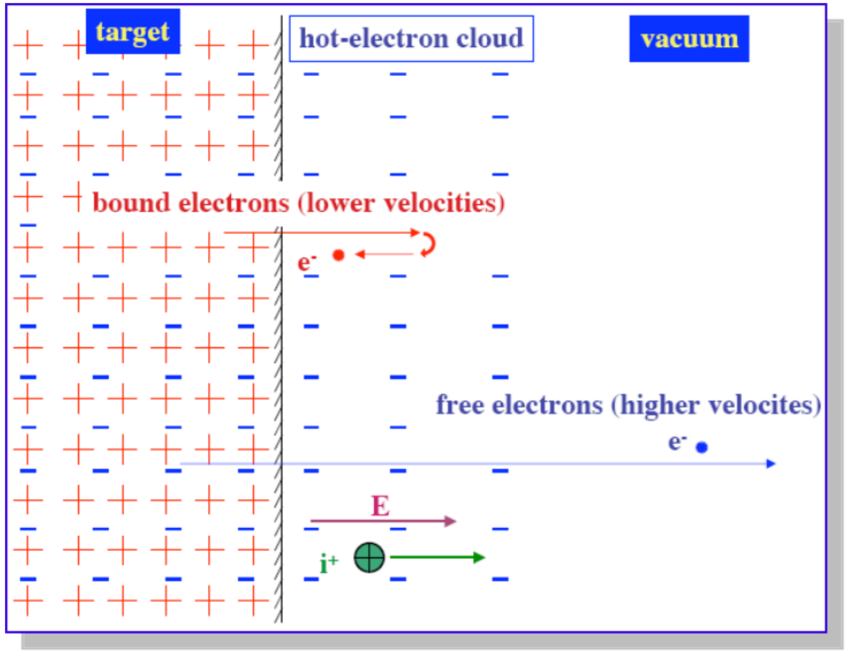
"… **A small fraction of the hot electron population escapes** and rapidly charges the target to a potential of the order of U_p preventing the bulk of the hot electrons from escaping. …"

"… All targets were **mounted on** 3 mm thick and 2 cm long **plastic stalks in order to provide a highly resistive path to the current flowing from the target to ground**. …"

about this issue, see also
K. Quinn et al. *Phys. Rev. Lett.* (2009)

Then, in usual conditions **a globally neutral target with only "bound" electrons develops**

1D 1T trapped electron model



$$f_e(x,p) = \frac{\tilde{n}}{2mcK_1\left(\frac{mc^2}{T}\right)} \exp\left(-\frac{\varepsilon(x,p)}{T_e}\right)$$

only the density of “trapped” e⁻ enters Poisson eq.; integrating over $\varepsilon < 0$ we get the trapped e⁻ density $n_{trap}(\phi(\mathbf{r}))$

$$n_{trap} = \int_{\varepsilon(\mathbf{r},\mathbf{p})\le 0} f(\mathbf{r},\mathbf{p})\, d^3p$$

$$\varepsilon(x,p) = m_e c^2[\gamma(p) - 1] - e\phi(x)$$

$$\Rightarrow \quad p^2 \le p_{max}^2 = m_e^2 c^2 \left[\left(\frac{e\phi}{m_e c^2}\right)^2 + \frac{2e\phi}{m_e c^2}\right]$$

The Poisson problem becomes

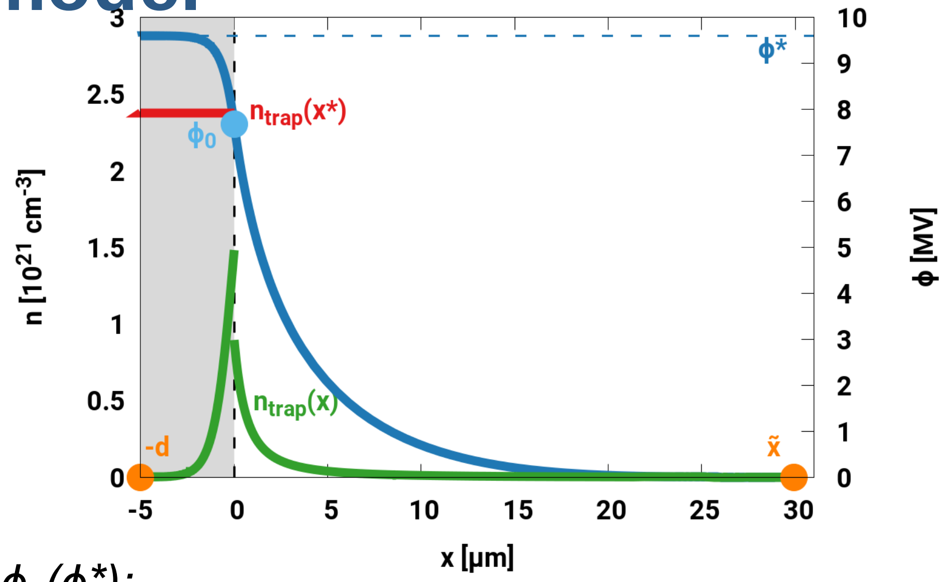
$$\begin{cases} \dfrac{\partial^2 \phi}{\partial x^2} = 4\pi e[n_{trap}(\phi(x)) - Zn_i] & x < 0 \\ \dfrac{\partial^2 \phi}{\partial x^2} = 4\pi e n_{trap}(\phi(x)) & x > 0 \end{cases}$$

Solve for $\phi(x)$ (BC are also required)

1D 1T trapped electron model

Model parameters:

- T_e
- ϕ^* = maximum value of ϕ (taken inside the target, at the left boundary condition)
- $\tilde{n}$ = reference density



we can find an implicit, analytical, relativistic solution

1. Set the parameters
2. Inside the target: integrate between ϕ^* and $\phi_0 = \phi(0)$ to get $\phi_0 = \phi_0(\phi^*)$:

$$\varphi_0 \approx \frac{(\varphi^* - 1)e^{\varphi^*} + 1}{e^{\varphi} - 1} \qquad \varphi = e\phi/T_e$$

3. Outside the target, x > 0, the implicit analytical relativistic solution is

$$\int_{\varphi(0)}^{\varphi(\xi)} \frac{d\varphi'}{\left(e^{\varphi'} I(\varphi') - e^{-\zeta}\beta\right)^{1/2}} = -\sqrt{2}\xi$$

$$I(\varphi) = \int_0^{\beta} e^{-\sqrt{\zeta^2+p^2}} dp \qquad \beta = \sqrt{(\varphi+\zeta)^2 - \zeta^2} \qquad \zeta = mc^2/T$$

the maximum ion energy can be estimated as

$$\mathcal{E}_{i,max} = Z\varphi_0 T_e$$

+ it is possible to compute the spectrum too

M. Passoni et al, *New J. Phys.* **12,** 045012 (2010)

1D 1T trapped electron model

How to set the model parameters?

- T_e →ponderomotive scaling
- ñ →usually fitted (it only affects the spatial scale, not the maximum proton energy)
- ϕ^* ? Note that: $e\phi^*$ = max trapped electron energy

- Make use of proper numerical simulations of laser-target interaction

- From the analysis of several published results
(starting with observed proton energies and using the model to infer ϕ^*)
we get the fitting (valid for the "ordinary" LC TNSA regime & optimal thickness)

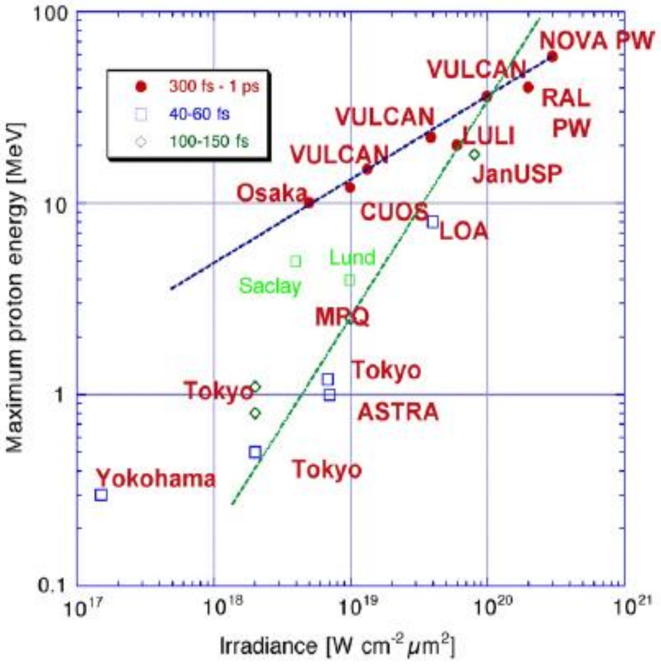
$$\boxed{\varphi^* = \frac{\varepsilon_{e,max}}{T_e} = A + B\log(E_L[\mathrm{J}])}$$

A=4.8, B=0.8, where E_L is the laser energy

$$\boxed{\mathcal{E}_{i,max} = Z\varphi_0(\varphi^*)T_e(I) = f(Z, E_L, I)}$$

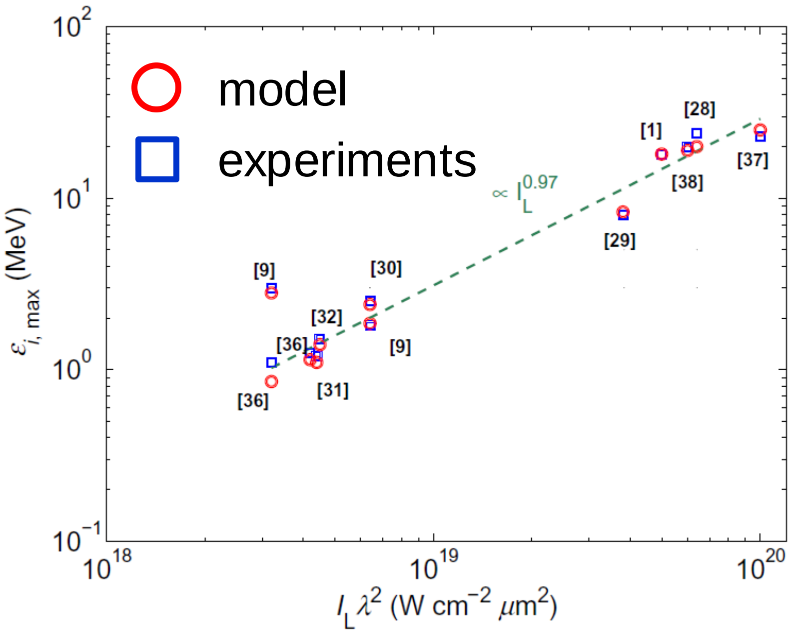
M. Passoni, M. Lontano, *Phys. Rev. Lett.,* 101, 115001 (2008)

1D 1T trapped electron model: comparison with experiments



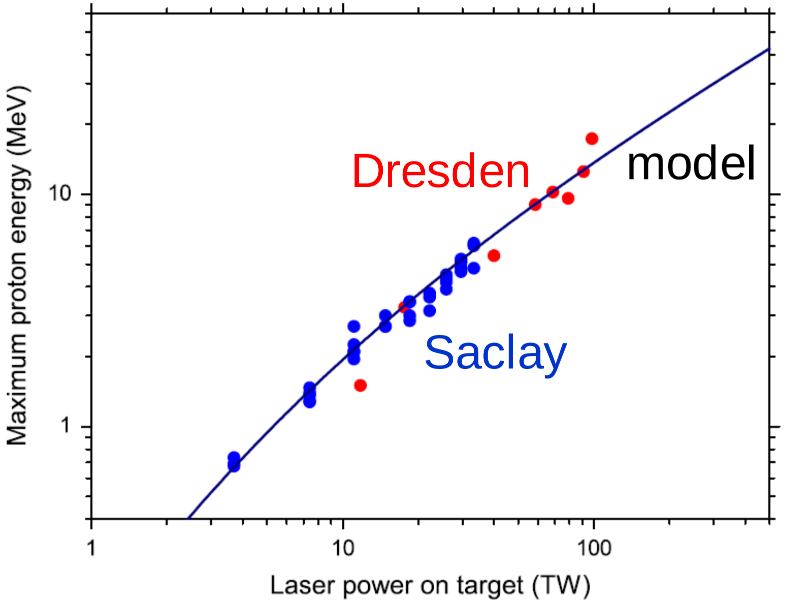
Experimental data from M. Borghesi et al., *Plasma Phys. Contr. Fus.* 50, 024140 (2008)

Effective linear dependence on I_L resulting from combined variation of pulse energy & intensity!



…agreement within 10 %

Good agreement also for fixed pulse duration (25 fs) and focal spot with UHC

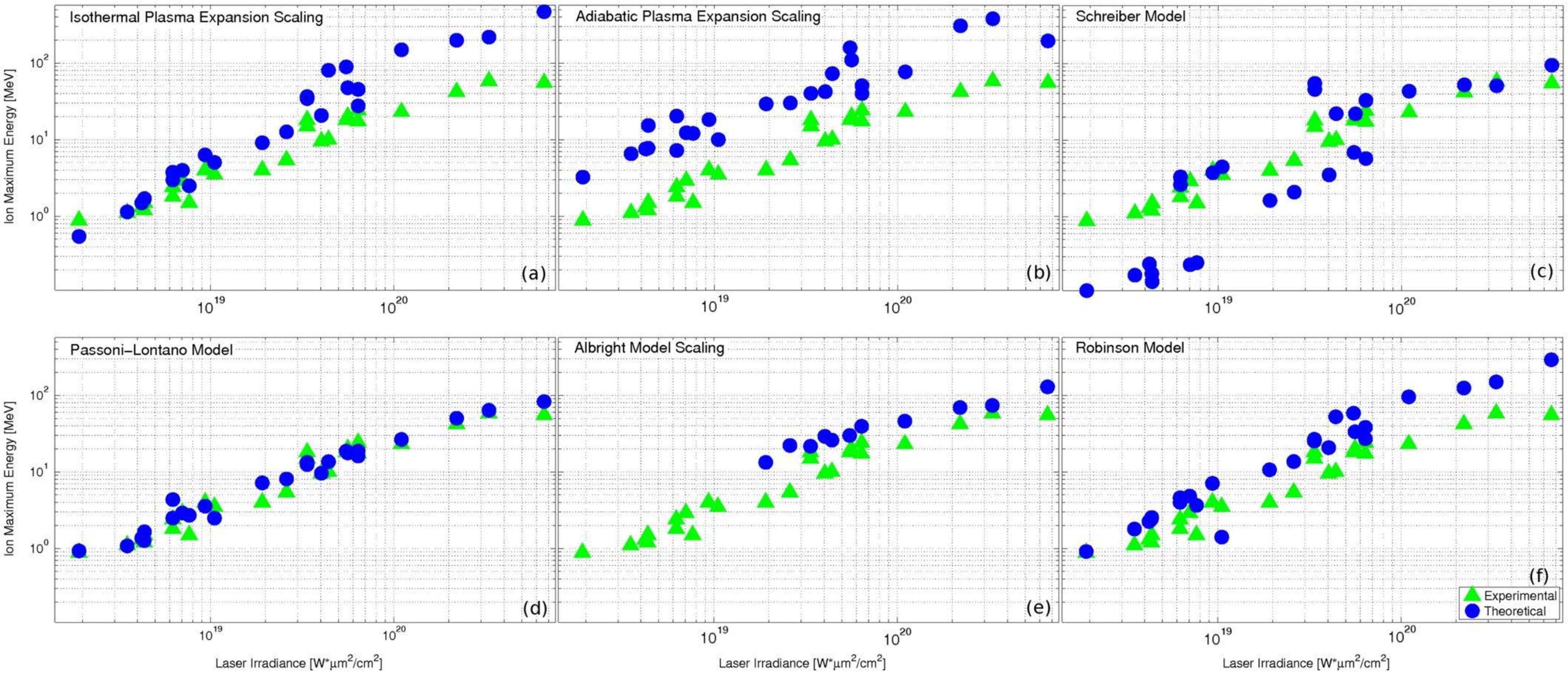


Saclay data:
T. Ceccotti et al, *Phys. Rev. Lett.* 99, 185002 (2007)
Dresden data:
K. Zeil et al., New J. Phys. 12, 045015 (2010)

M. Passoni et al, *New J. Phys.* **12** 045012, (2010) A. Zani, *Nucl. Instr. and Meth. A* **653**, 89 (2011)

Comparisons between different TNSA models

Quasi static models appear to be an effective tool to describe, simply but effectively, the maximum energy observed in TNSA experiments



C. Perego, et al., Nucl. Instr. and Meth. A 653 , 94-97 (2011)

Limits of 1T models & 2T model

at least two electron populations exist:

1) **"hot" (fast, *h*), low density, laser-generated:**
e-density ~ critical density ~ 10^{20}-10^{21} cm^{-3}
T_h ~ ponderomotive potential ~ few MeVs

2) **"cold" (bulk, *c*), high density, ohmically heated, e- pop.**
density ~ solid density ~ 10^{23}-10^{24} cm^{-3}
T_c= ? …> T_{room}

Both populations affect the properties of the self-consistent electric field on the rear surface ---> effects on ion acceleration

"*Charge separation effects in solid targets and ion acceleration with a two-temperature electron distribution*", M. Passoni, *et al*., *Phys. Rev. E* 69, 026411 (2004)

introduce → find implicit solution

$$a = \frac{N_{0c}}{N_{0h}} \qquad b = \frac{T_c}{T_h}$$

$$\lambda_{Dh} = \sqrt{\frac{T_h}{4\pi N_{0h} e^2}} \qquad \varphi = \frac{e\phi}{T_h}$$

inside the target
$$\int_{\varphi(0)}^{\varphi(x)} \frac{d\varphi}{[\exp\varphi + ab\exp(\varphi/b) - (1+ab) - (1+a)\varphi]^{1/2}} = -\sqrt{2}\frac{x}{\lambda_{Dh}}$$

outside the target
$$\int_{\varphi(0)}^{\varphi(x)} \frac{d\varphi}{[\exp\varphi + ab\exp(\varphi/b)]^{1/2}} = -\sqrt{2}\frac{x}{\lambda_{Dh}}$$

2T model: ion acceleration

max value of electric field close to target higher than in 1-T

----> it can affect acceleration

the peak in E influences the proton trajectory

----> application as an “injector”

influence on the so-called “double-layer” acceleration scheme

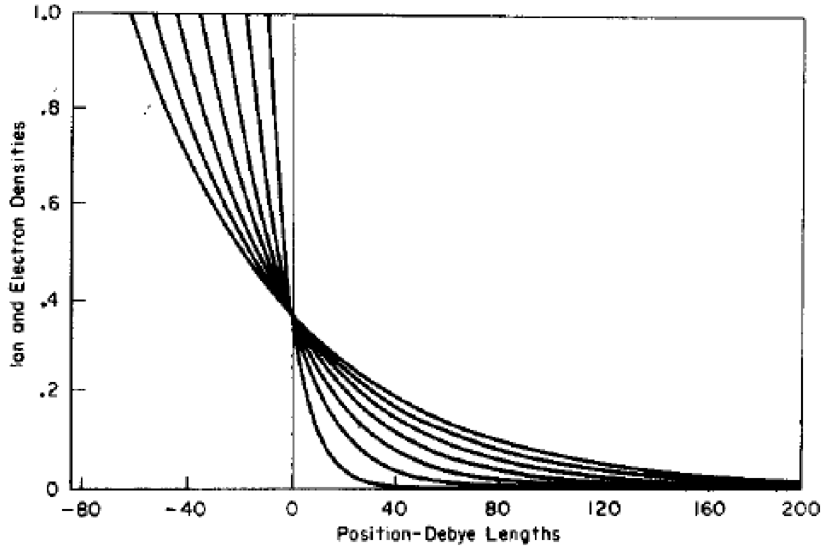
----> *E* peak is close to the proton layer

penetration of the electric field inside the target is determined by the cold electrons

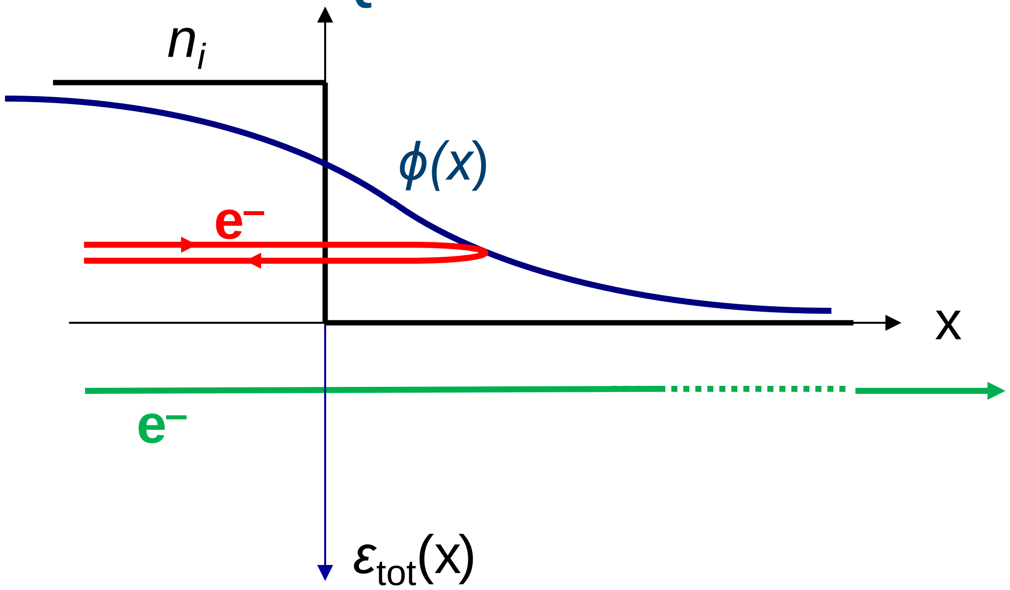
the effect of the background e- population is important
in the description of the rear acceleration mechanism

Main analytical approaches: conclusions

Plasma expansion



Quasi-static



- give simple expressions for the maximum ion energy and energy spectrum
- quite good agreement with experimental results

- capture the long-time evolution
- can describe the spectral evolution of the accelerated ions
- describe best the acceleration driven by long pulses

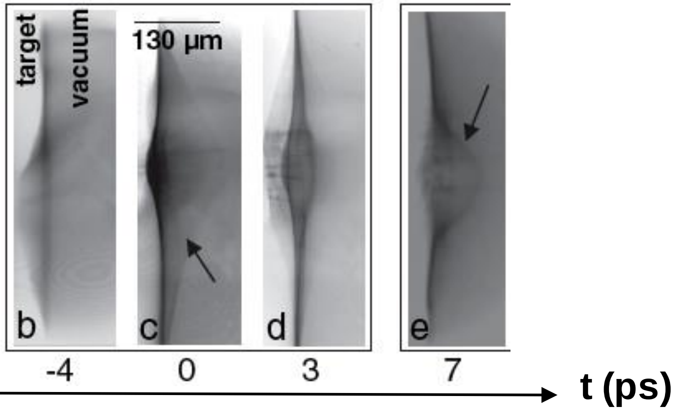
- capture the early-stage dynamics
- most effective to determine the properties of the highest-energy ions, especially the maximum energy
- especially appropriate for ultrashort laser pulses

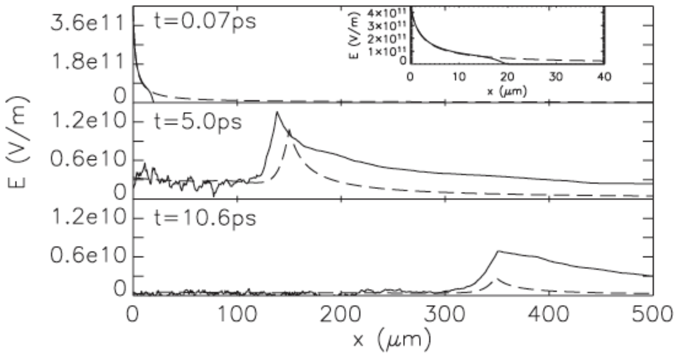
What else?

Numerical simulations

- **Analytical descriptions are very useful but oversimplified** and often limited to specific configurations
- **Numerical simulations**, in principle, provide more general tools, i.e. can be **easily adapted to many different situations**
  - Complex configurations can be simulated (e.g. 2D, 3D, non-uniform density profiles, etc.)
  - Different mechanisms and regimes can be explored
  - Easier than performing some type of experiments, especially in wide parametric scans
- **Support experimental observations** (including the early laser-ion acceleration experiments from years ~ 2000)

An example: experimental data best reproduced by PIC simulations assuming a zero field at a finite distance h ≈ 20 μm from the rear surface





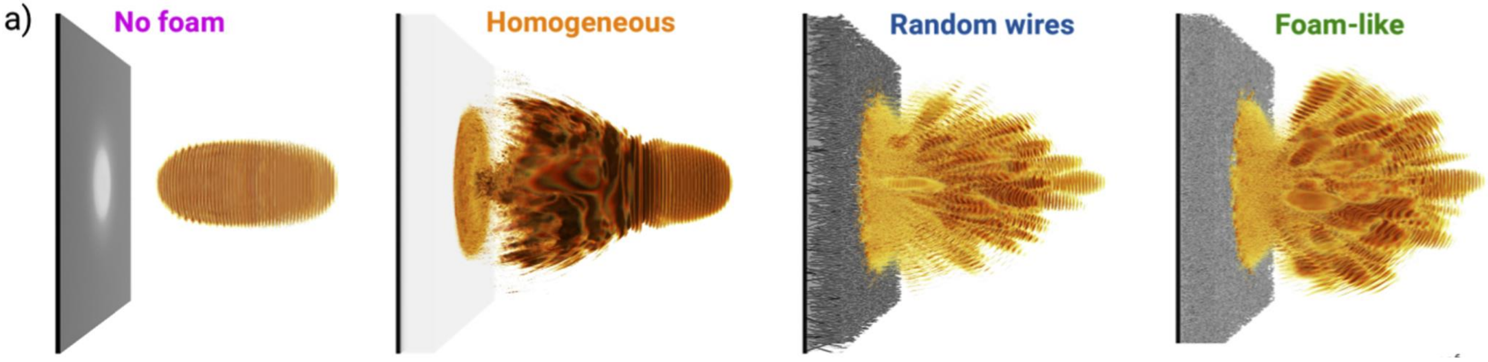
By far most-established numerical tool: particle-in-cell (PIC) simulations

! See Dimitriou's lecture

L. Romagnani, *et al*., *Phys. Rev. Lett.* 95, 195001 (2005)

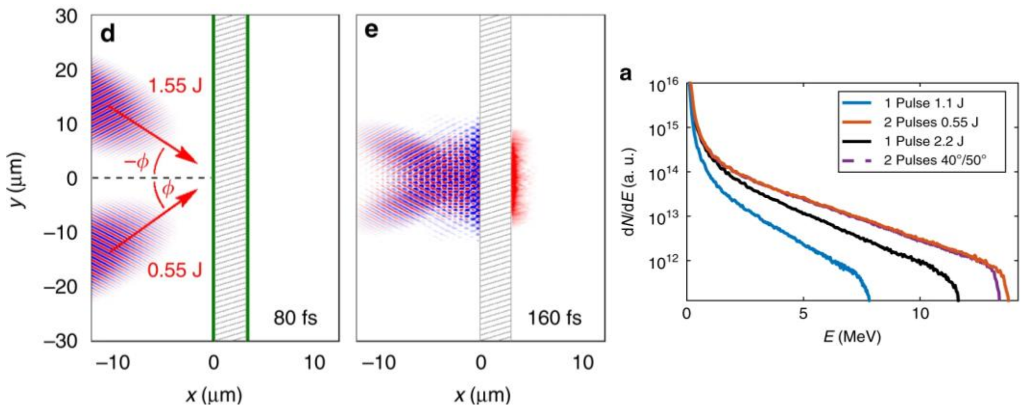
Nowadays PIC simulations give access to extremely complex scenarios

Nanostructured targets



Passoni et al., *Plasma Phys. Control. Fusion* 62.1 (2019): 014022.

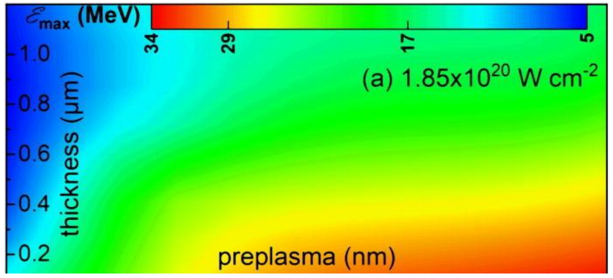
Multiple laser pulses



Ferri et al., *Comm. Physics* 2.1 (2019): 1-8.

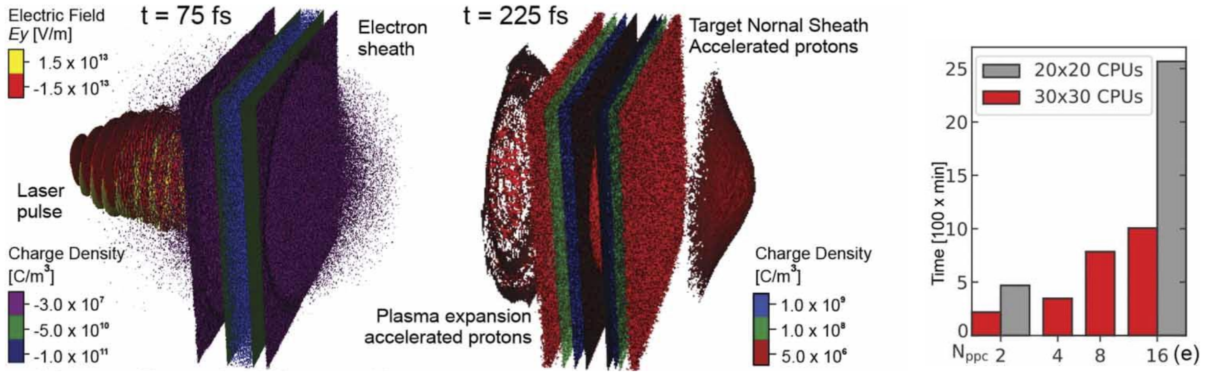
Prepulse effects with hydrodynamic codes

hydrodynamic simulations give the target density after the ~ns ASE pedestal and prior to main pulse, which then is fed to PIC



Hadjisolomou et al., *Phys. Plasmas* 27.1 (2020): 013107.

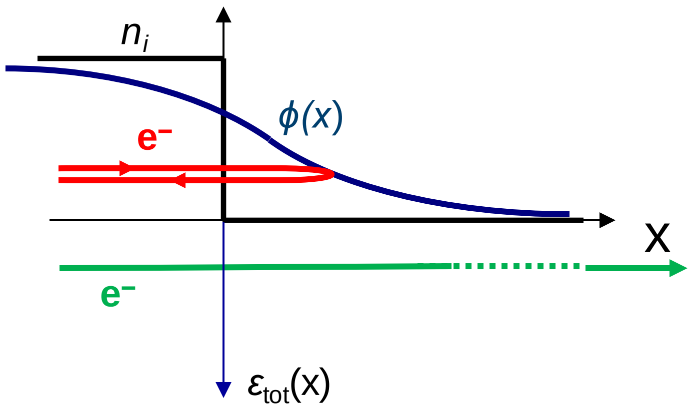
Performance tests

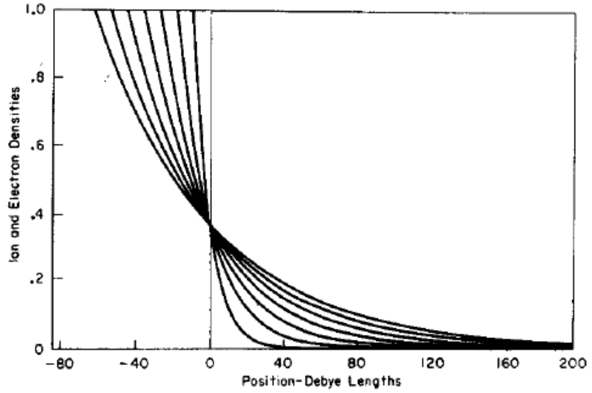


Tazes et al., *Plasma Phys. Control. Fusion* 62.9 (2020): 094005.

Theoretical approaches: conclusions

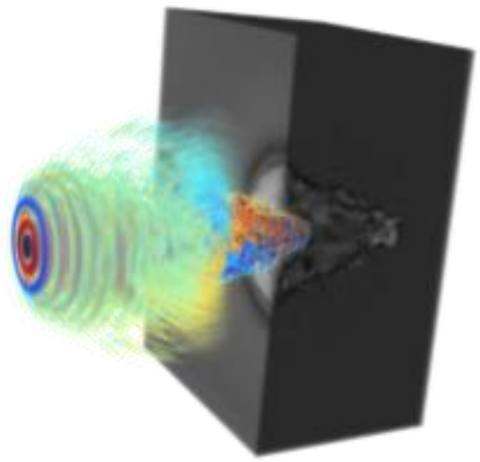
Analytical models





– Highly simplified
– Highly specific
+ Easy to use and manage
+ Provide the essential physics

PIC simulations



+ Highly versatile and adaptable
+ Virtually any regime available
– Highly complex results
– Computational constraints

Overall, numerical simulations and analytical models are complementary tools

Lecture conclusions

- LASER-DRIVEN ION ACCELERATION is a complex physical process:
  - laser-solid interaction and conversion into energetic electrons
  - electron transport in the solid target
  - development of strong charge separation and huge accelerating fields

- TNSA IS THE MOST DOMINANT PROCESS in present experimental conditions

- for proper TNSA modeling, QUASI-NEUTRALITY CANNOT BE ASSUMED

- SINGLE COMPONENT PLASMA EXPANSION provides first useful insight

- QUASI-STATIC APPROACH seems promising for effective TNSA description

- both plasma expansion models and quasi-static models can provide simplified yet useful insights in the ion acceleration process and can be COMPLEMENTARY

Thank you for the attention!

Bibliography: more on plasma expansion models

"*Self-similar expansion of a plasma into a vacuum*"
P. Mora, R. Pellat, *Phys. Fluids* 22, 2300 (1979)
- extension of previous theories to non-equilibrium electron distribution function (first order in $(Zm/M)^{1/2}$); energy conservation

"*Exact solution of Vlasov equations for quasi-neutral expansion of plasma bunch into vacuum*"
D.S. Dorozhkina, V.E. Semenov, *Phys. Rev. Lett.* 81, 2691 (1998)
- quasi-neutral approximation: electron and ion expansion in the presence of self-consistent electric field;
- arbitrary el-ion mass-ratio Zm/M, T_e/T_i, f_e, three-dimensional

"*Particle dynamics during adiabatic expansion of a plasma bunch*"
V.F. Kovalev, *et al*., *JETP* 95, 226 (2002)
- quasi-neutral approximation: renormalization-group approach → adiabatic expansion for arbitrary distribution functions;
- used for a 2-temperature e.d.f., and different ion species, one-dim.

Bibliography: other quasi-static models

"*Analytical Model for Ion Acceleration by High-Intensity Laser Pulses* "
J. Schreiber, *et al*. *Phys. Rev. Lett*. 97, 045005 (2006)

- surface charge model exploiting radial symmetry for the electric field

"*Theory of Laser Acceleration of Light-Ion Beams from Interaction of Ultrahigh-Intensity Lasers with Layered Targets* "
B.J. Albright, *et al*., *Phys. Rev. Lett*. 97, 115002 (2006)

- extention to describe layered targets

"*The laser proton acceleration in the strong charge separation regime* "
M. Nishiuchi, *et al*., *Phys. Lett. A* 357, 339 (2006)

- approach analogous to the 1T model to interpret experiments

" *Effect of Target Composition on Proton Energy Spectra in Ultraintense Laser-Solid Interactions* "
A.P. Robinson, *et al*., *Phys. Rev. Lett.* 96, 035005 (2006)

- study of the effects of a non negligible proton density in the target

Bibliography: further theoretical references

"*Ion acceleration in expanding multi-species plasmas*"
V. Yu. Bychenkov *et al*., *Phys. Plasmas,* 11, 3242 (2004)

"*Ion acceleration in short-laser-pulse interaction with solid foils* "
V. T. Tikhonchuk, *et al*., *Plasma Phys. Controlled Fusion* 47, B869 (2005)

"*Collisionless expansion of a Gaussian plasma into a vacuum*"
P. Mora, *Phys. Plasmas* 12, 112102 (2005)

"*Thin-foil expansion into a vacuum*"
P. Mora, *Phys. Rev. E* 72, 056401 (2005)

"Test ion acceleration in a dynamic planar electron sheath "
M.M. Basko, *Eur. Phys. J. D,* 41, 641 (2007)

"*Nanocluster explosions and quasimonoenergetic spectra by homogeneously distributed impurity ions*"
M. Murakami & M. Tanaka, *Phys. Plasmas* 15, 082702 (2008)